

MECH 3805
Winter 2017

QUIZ 1.A
Monday Group

Name: _____

Student No: _____

750 g of oxygen gas at an initial pressure of 1 MPa and initial temperature of 65°C expands in a piston-cylinder device following the relationship $Pv^{1.15} = \text{constant}$ until its volume doubles. Draw a schematic of the process, state your assumptions, and find V_1 , V_2 , P_2 , T_2 and W_{1-2} . ($R_{\text{Oxygen}} = 0.26 \text{ kJ/kg.K}$)

MECH 3805
Winter 2017

QUIZ 1.B
Monday Group

Name: _____

Student No: _____

750 g of oxygen gas at an initial pressure of 1 MPa and initial temperature of 75°C expands in a piston-cylinder device following the relationship $Pv^{1.15} = \text{constant}$ until its volume doubles. Draw a schematic of the process, state your assumptions, and find V_1 , V_2 , P_2 , T_2 and W_{1-2} .

SOLUTION 1.A

Assumptions:

Air is an ideal gas

Piston is leak proof and frictionless

$$P_1V_1 = mRT_1$$

$$R_{\text{Oxygen}} = 0.26 \text{ kJ/kg.K}$$

$$V_1 = [(0.75 \text{ kg})(0.26 \text{ kJ/kg.K})(273+65)\text{K}] / (1,000 \text{ kPa}) = 0.066 \text{ m}^3$$

$$V_2 = 2V_1 = 0.132 \text{ m}^3$$

$$P_2 = P_1(V_1/V_2)^{1.15} = (1,000 \text{ kPa})(1/2)^{1.15} = 451 \text{ kPa}$$

$$T_2 = (P_2V_2/mR) = [(451 \text{ kPa})(0.132 \text{ m}^3)] / [(0.75 \text{ kg})(0.26 \text{ kJ/kg.K})] = 305 \text{ K}$$

$$W_{1-2} = (P_2V_2 - P_1V_1)/(1-n) \quad (\text{eq. 2.16c in textbook})$$

$$W_{1-2} = [(451 \text{ kPa})(0.132 \text{ m}^3) - (1,000 \text{ kPa})(0.066 \text{ m}^3)] / (1-1.15) = 43.1 \text{ kJ}$$

SOLUTION 1.B

Assumptions:

Air is an ideal gas

Piston is leak proof and frictionless

$$P_1V_1 = mRT_1$$

$$R_{\text{Oxygen}} = 0.26 \text{ kJ/kg.K}$$

$$V_1 = [(0.75 \text{ kg})(0.26 \text{ kJ/kg.K})(273+75)\text{K}] / (1,000 \text{ kPa}) = 0.068 \text{ m}^3$$

$$V_2 = 2V_1 = 0.136 \text{ m}^3$$

$$P_2 = P_1(V_1/V_2)^{1.15} = (1,000 \text{ kPa})(1/2)^{1.15} = 451 \text{ kPa}$$

$$T_2 = (P_2V_2/mR) = [(451 \text{ kPa})(0.136 \text{ m}^3)] / [(0.75 \text{ kg})(0.26 \text{ kJ/kg.K})] = 315 \text{ K}$$

$$W_{1-2} = (P_2V_2 - P_1V_1)/(1-n) \quad (\text{eq. 2.16c in textbook})$$

$$W_{1-2} = [(451 \text{ kPa})(0.136 \text{ m}^3) - (1,000 \text{ kPa})(0.068 \text{ m}^3)] / (1-1.15) = 44.4 \text{ kJ}$$

A jet of air with a mass flow rate of 10 kg/s, temperature of 300°C, velocity of 300 m/s, pressure of 500 kPa and an enthalpy of 800 kJ/kg enters a turbine. The potential energy of the air is negligible. $R_{\text{air}} = 0.29 \text{ kJ/kg.K}$

- Draw a schematic of the system, state your assumptions
- Find the volume flow rate of the air and the rate of energy transfer to the system via the mass flow of the air.
- Does the negligible potential energy statement make sense? Discuss quantitatively.

MECH 3805
Winter 2017

QUIZ 1.B
Friday Group

Name: _____

Student No: _____

Air with a mass flow rate of 10 kg/s, temperature of 300°C, velocity of 400 m/s, pressure of 500 kPa and an enthalpy of 800 kJ/kg enters a turbine. The potential energy of the air is negligible. $R_{\text{air}} = 0.29 \text{ kJ/kg.K}$

- Draw a schematic of the system, state your assumptions
- Find the volume flow rate of the air and the rate of energy transfer to the system via the mass flow of the air.
- Does the negligible potential energy statement make sense? Discuss quantitatively.

Solution 1.A

Assumption: air is an ideal gas

$$P_1 \dot{V}_1 = \dot{m} R T_1$$

$$(500 \text{ kPa})(\dot{V}_1) \frac{\text{m}^3}{\text{s}} = \left(10 \frac{\text{kg}}{\text{s}}\right) \left(0.29 \frac{\text{kJ}}{\text{kgK}}\right) (300 + 273) \text{K}$$

$$\dot{V}_1 = 3.3 \text{ m}^3/\text{s}$$

$$\dot{E}_{massflow} = \dot{m} \left(h + \frac{1}{2} \vec{V}^2 + gz \right)$$

$$\dot{E}_{massflow} = (10 \text{ kg/s})(800,000 \text{ J/kg} + (\frac{1}{2})(300 \text{ m/s})^2 + 0) = 8,450,000 \text{ J/s} = 8.45 \text{ MW}$$

Most human settlements are at altitudes less than 1,000 m. Using $g=10 \text{ m/s}^2$, the potential energy term would have a magnitude of about 10,000 J/kg. Compared to the sum of h and ke terms (800,000 J/kg + 45,000 J/kg), this is about 1%. So, it's ok to neglect.

Solution 1.B

Assumption: air is an ideal gas

$$P_1 \dot{V}_1 = \dot{m} R T_1$$

$$(500 \text{ kPa})(\dot{V}_1) \frac{\text{m}^3}{\text{s}} = \left(10 \frac{\text{kg}}{\text{s}}\right) \left(0.29 \frac{\text{kJ}}{\text{kgK}}\right) (300 + 273) \text{K}$$

$$\dot{V}_1 = 3.3 \text{ m}^3/\text{s}$$

$$\dot{E}_{massflow} = \dot{m} \left(h + \frac{1}{2} \vec{V}^2 + gz \right)$$

$$\dot{E}_{massflow} = (10 \text{ kg/s})(800,000 \text{ J/kg} + (\frac{1}{2})(400 \text{ m/s})^2 + 0) = 8,800,000 \text{ J/s} = 8.8 \text{ MW}$$

Most human settlements are at altitudes less than 1,000 m. Using $g=10 \text{ m/s}^2$, the potential energy term would have a magnitude of about 10,000 J/kg. Compared to the sum of h and ke terms (800,000 J/kg + 80,000 J/kg), this is about 1%. So, it's ok to neglect.

MECH 3805
Winter 2017

QUIZ 1.C
Monday Group

Name: _____

Student No: _____

A piston-cylinder device with a diameter of 1 m and height (from the bottom of the piston to the bottom of the cylinder) of 1 m contains air at 20°C. The mass of the piston is 500 kg, and the acceleration due to gravity is 10 m/s². The local atmospheric pressure is 100 kPa. $R_{\text{air}}=0.287 \text{ kJ/kg}\cdot\text{K}$

Draw a schematic, state your assumptions and determine:

- the mass and specific volume of the air in the cylinder,
- the mass of a set of weights that needs to be added to the top of the piston so that the absolute pressure of the air in the cylinder is 250 kPa.

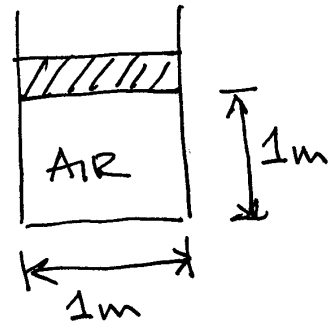
$$D = 1\text{m}$$

$$H = 1\text{m}$$

$$T = 20^\circ\text{C} = 293\text{K}$$

$$m_{\text{PISTON}} = 500\text{kg} \quad g = 10\text{m/s}^2$$

ASSUME:
FRICTIONLESS
PISTON



$$A_{\text{piston}} = \frac{\pi D^2}{4} = 0.785\text{m}^2$$

$$V_{\text{Piston-cylinder}} = A \cdot H = 0.785\text{m}^3$$

$$P_{\text{ATM}} = 100\text{kPa} \quad R_{\text{AIR}} = 0.287\text{kJ/kg}\cdot\text{K}$$

$$PV = mRT \quad P_{\text{DUE TO MASS}} = \frac{F}{A} = \frac{m \cdot g}{A} = \frac{500\text{kg} \cdot 10\text{m/s}^2}{0.785\text{m}^2} = 6369\text{Pa} = 6.4\text{kPa}$$

$$P_{\text{AIR}} = P_{\text{ATM}} + P_{\text{DUE TO MASS}} = 106.4\text{kPa}$$

$$m = \frac{PV}{RT} = \frac{106.4\text{kPa} \times 0.785\text{m}^3}{0.287\text{kJ/kg}\cdot\text{K} \times 293\text{K}} = 0.99\text{kg} \approx \underline{\underline{1\text{kg}}}$$

$$v = \frac{V}{m} = \underline{\underline{0.785\text{m}^3/\text{kg}}}$$

$$P_{\text{FINAL}} = 250\text{kPa} \quad P_{\text{DUE TO NEW MASS}} = 250 - 106.4 = 143.6\text{kPa}$$

$$m_{\text{NEW MASS}} = \frac{PA}{g} = \frac{143600\text{Pa} \times 0.785\text{m}^2}{10\text{m/s}^2} = \underline{\underline{11,273\text{kg}}}$$